

**The answer:****Total marks (20 marks)****Question No. 1****(6 marks)****1. Write a regular expression for the following languages.****(4 marks)****a) All strings of p's and q's which contains an odd number of q's. (2 marks)**

$$p^* q (q p^* q | p)^*$$

**b) All strings of a's and b's that contain no three consecutive b's. (2 marks)**

$$(\epsilon | b | bb)(abb | ab | a)^*$$

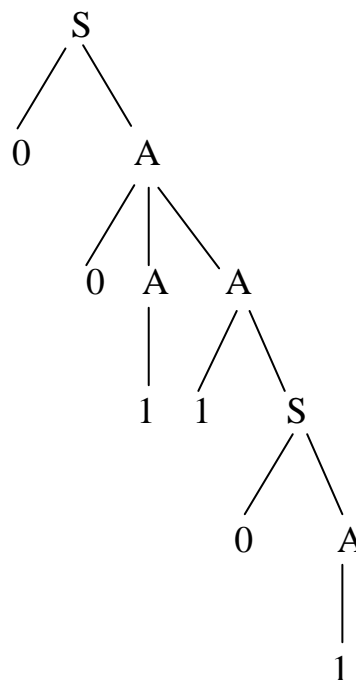
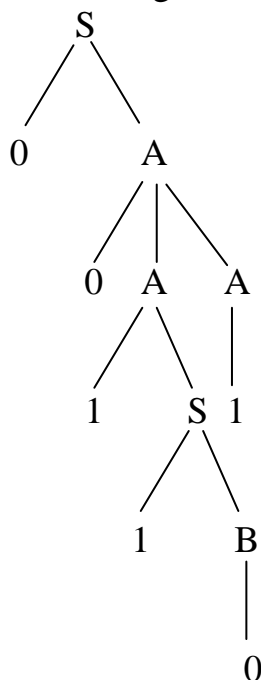
**2. Explain why the grammar below is ambiguous.****(2 marks)**

$$S \rightarrow 0A \mid 1B$$

$$A \rightarrow 0AA \mid 1S \mid 1$$

$$B \rightarrow 1BB \mid 0S \mid 0$$

The grammar is ambiguous because we can find strings that have multiple parse trees. The string 001101 has two distinct parse trees.



$$S \Rightarrow 0A \Rightarrow 00AA \Rightarrow 001S1 \Rightarrow 0011B1 \Rightarrow 001101$$

$$S \Rightarrow 0A \Rightarrow 00AA \Rightarrow 0011S \Rightarrow 00110A \Rightarrow 001101$$

## Question No. 2

(6 marks)

1. Consider the regular expression below which can be used as part of a specification of the definition of exponents in floating-point numbers. Assume that the alphabet consists of numeric digits ('0' through '9') and alphanumeric characters ('a' through 'z' and 'A' through 'Z') with the addition of a selected small set of punctuation and special characters (say in this example only the characters '+' and '-' are relevant). Also, in this representation of regular expressions the character '.' denotes concatenation.

$$\text{Exponent} = (+ | - | \epsilon) \cdot (E | e) \cdot (\text{digit})^+$$

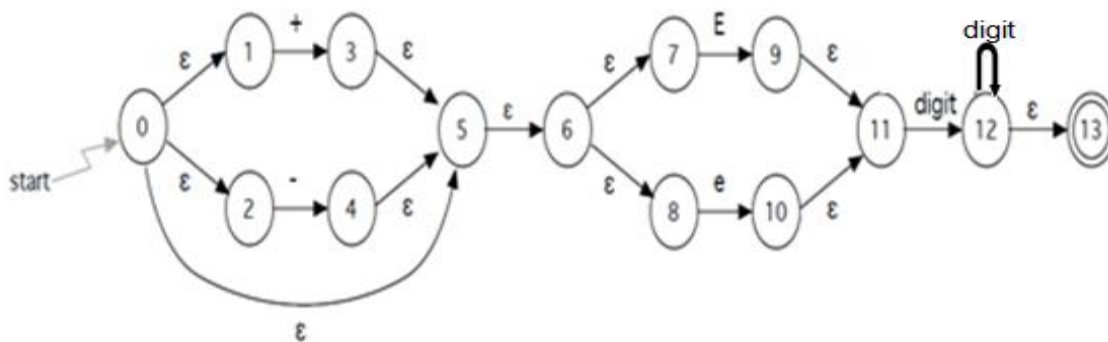
For this regular expression answer the following questions:

(4 marks)

- a) Derive an NFA capable of recognizing this language.

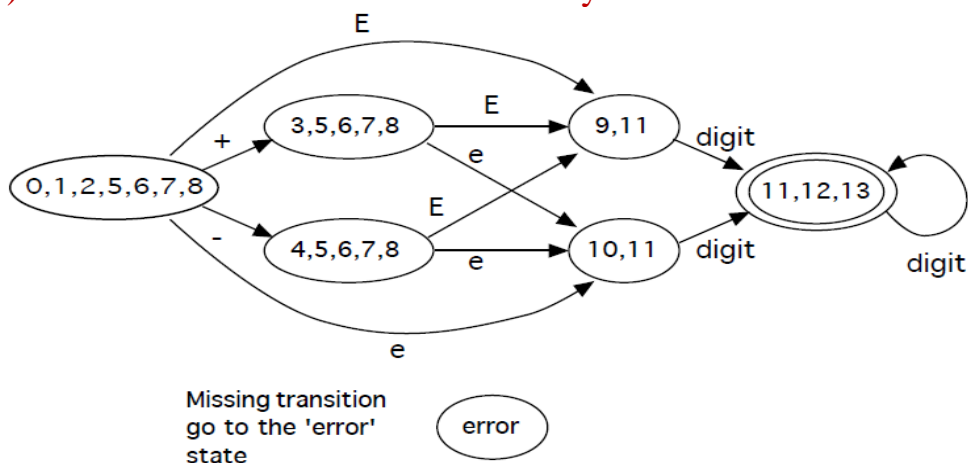
(2 marks)

After some basic simplifications, which consist mostly on the elimination of consecutive  $\epsilon$ -transitions, we would arrive at the following NFA.

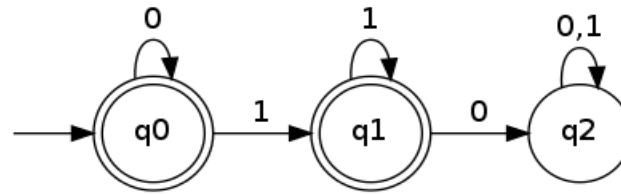


- b) Derive the DFA for the NFA that you derive in a.

(2 marks)



2. Consider the following deterministic finite automata over the alphabet  $\Sigma=\{0,1\}$ . (2 marks)



i. Give a one-sentence description of the language recognized by the DFA. (1 marks)

The language of all the strings from the given alphabet  $\Sigma=\{0,1\}$  that contains zero or more number of 0's followed by zero or more number of 1's.

ii. Write a regular expression for this language. (1 marks)  
 $0^*1^*$

**Question No. 3** (8 marks)

1. Given the following ambiguous context free grammar

$S \rightarrow Ab \mid aaB$

$A \rightarrow a \mid Aa$

$B \rightarrow b$

(a) Show that the string  $s = aab$  has two leftmost derivations. (2 marks)

The string  $s = aab$  has the following two leftmost derivations

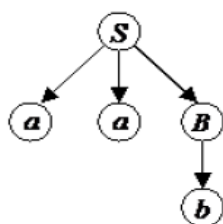
$S \Rightarrow aaB \Rightarrow aab$

$S \Rightarrow AB \Rightarrow AaB \Rightarrow aaB \Rightarrow aab$

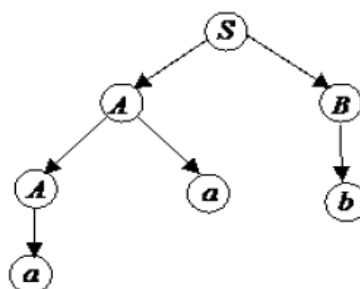
(b) Show the two derivation trees for the string  $s$ . (2 marks)

The two derivation trees of string  $aab$  are shown below.

$S \Rightarrow aaB \Rightarrow aab$



$S \Rightarrow AB \Rightarrow AaB \Rightarrow aaB \Rightarrow aab$



(c) Find an equivalent unambiguous context-free grammar. (2 marks)

The equivalent unambiguous grammar is following

$S \rightarrow Ab$

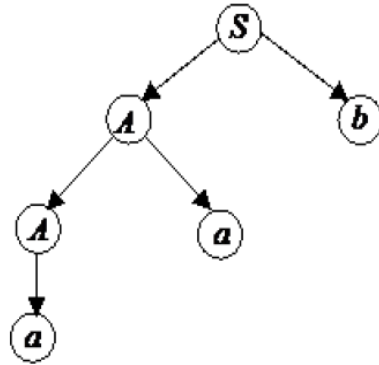
$A \rightarrow a \mid Aa$

This grammar is not ambiguous because at any derivation step there is only one choice to make. This grammar is equivalent to the previous one, because both grammars generate the same language: all the strings that start with one or more a, and end with a single b.

(d) Give the unique leftmost derivation and derivation tree for the string  $s$  generated from your rewritten unambiguous grammar. (2 marks)

With the new grammar the unique leftmost derivation and derivation tree of the string  $aab$  are shown below.

$$S \Rightarrow Ab \Rightarrow Aab \Rightarrow aab$$



*Best wishes*

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